

Sensitivity analysis of bread production

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Abstract

Keywords

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A system could be any of activities within an organization, from production to a sales operation. In all cases limited resources are allocated for achievement of basic objectives. The objective might be to maximize profit or minimize cost. This paper makes use of data from OBA 'B' Bakery Industry Offa, Kwara State. Which includes machine hours for cutting, number of laborers', quality of water used, oven (heat) temperature and bags of flours used for production of bread. Sensitivity Analysis was employed, it was discovered that when the price of both basic and non-basic products changed the objectives function also changed and the total profit increased. It was established that the use of electrical oven improved the production and required less labour, time and maximize profit.

Introduction

Linear programming is a mathematical technique that is concern with allocation of scarce resources. Linear programming is also a mathematical technique for finding the best uses of organization resources and its technique for choosing the best alternative from a set of feasible alternatives.

The study of the effect of changes in the parameter of the problem in the current optimal solution after a Linear programming problem has been solved is defined as sensitivity analysis. It allows one to take decisions on how to change cost coefficients, change right coefficients of constraints, addition of a new variable, addition of

new variables and deletion of a variable (Nilu, Ahmed and Bhuiyan, 2017). It plays a crucial role in the assessment of different parameters in LP problems. It is also known as post optimality analysis.

Sensitivity analysis is also the study of how the uncertainty in the output of a mathematical model or system can be divided to different sources of uncertainty in its inputs.

Inflation is typically a broad measure, such as the overall increase in prices or the increase in the cost of living in a country. It is the rate of increase in prices over a given period of time. This portends a decrease in the purchasing power of

money, resulting in a general increase of prices of goods and services. Despite investment of billions in agriculture, Nigeria's food inflation rate has been up since mid 2015 to date. According to Nairametrics report of 2022, the rise in the food index was caused by 'increases' in the prices of bread and cereals, food product, potatoes, yam and other tubers, soft drinks, fruits, oils and fats. To this end, one begins to wonder if certain range of prices variables will eventually give desirable predicted outcomes (in the face of erratic changes in prices) The aim of this study is to apply sensitivity analysis to model bread production while the specific objectives are to: determine appropriate constraints which will maximize the company's profit, examine effect of changes in cost variable(s) on the optimality solution and examine these changes, identify changes in key variables which mostly influence the production cost and determine changes in the objective function (i.e., profit of different size of bread as the result of changes in variable).

The work of Nilu, Ahmed and Bhuiyan (2017) provide a great motivation for this work. They implemented the concept of sensitivity analysis of Linear Programming Problems (LPPs) in real life situation by developing a computer code using LINDO (Linear, Interactive and Discrete Optimizer). Their work motivated this research which intends to apply sensitivity analysis to bread production using TORA (Temporary Ordered Routing Algorithm). Allocation of scarce resources has been one of man's oldest problems. Resource allocation becomes great challenge when there are various demands and supply to be met. Hence, this study will provide a means of identifying the parameter which optimizes the objective value (profit). It will also, provide a means of obtaining precise estimate of these parameters, thereby, increasing the reliability of the model as well as the solution.

Considered the modeling local gas production rate in municipal landfills using local mechanistic reaction model. They analysed the sensitivity of the model to changes in each of their 40 input parameters. The results of their sensitivity analysis suggest that the parameters with the

largest impact on model output were the hydrolysis constant, the kinetic constants, the water content, temperature, pH, initial concentration, enthalpy of the original organic matter as well as the protein content and the buffering capacity of the waste.

Sanchez et al. (2012) Through the example of apartment building, they applied sensitivity analysis in building energy simulation (model). They analysed the potential usefulness of combining first and second-order sensitivity analysis using an elementary effects method by including an analysis of interactions between-the-input parameters(second-order analysis). They proposed solutions to differentiate non-linearity from higher order interaction.

Suresh (2012) Studied application of sensitivity analysis in capital budgeting decisions. He posited that the main aim of every investment is to get proper yield from the project, so that it can recover the cost associated with each source of funds and earn required amount of profit to compensate the risk involved in the business. Also, he stated that all these costs must be recovered through judicious allocation of available fund. He concluded that investment decision are based on risk and uncertainty.

Khan, Hashmi, Hussain and Ilyas (2017) conducted a sensitivity analysis for the determinants of investment appraisal of Pakistani non-financial firms listed at Pakistan Stock Exchange (PSX) across sectors. They employed OLS regression along with common effect and fixed effect model on panel data pertaining to 60 non-listed firms at PSX over the period 2003 to 2015. Results showed that leverage, growth, dynamism, and inflation have strong positive associations with investment appraisal, however, munificence and GDP influence the process conversely

Materials and Method

Allocation of scarce resources has been one of man's oldest problems. Resource allocation becomes great challenge when there are various demands and supply to be met.

Hence, this study will provide a means of identifying the parameter which optimizes the objective value (profit). It will also, provide a means of obtaining precise estimate of these parameters, thereby, increasing the reliability of the model as well as the solution.

In this work, the application of sensitivity analysis to the production of bread at OBA "B" Bakery, Offa, Kwara State is considered. TORA was used to analyse the data of daily bread production.

Data used in this work is a secondary data. It was collected from OBA 'B' Bread Baking Factory, Offa, Kwara State. The data include machine hour for cutting, number of labours, quantity of water used, oven (heat) temperature and bags of flour used for daily production of breads.

General Forms of Linear Programming Problem

Linear Programming Problem (LPP) can be classified into two forms namely;

-) Maximization Problem
-) Minimization Problem
-) **Maximization Problem**

For instance, if we are to maximize Z:

$$\begin{aligned} \text{Max } Z &= c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n \text{ (Objective function)} \\ \text{Subject to (s.t)} \quad &a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n \leq b_1 \\ &a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n \leq b_2 \\ &a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n \leq b_m \end{aligned}$$

$x_i \geq 0, i = 1, 2, 3, \dots, n$ i.e. non-negative constraint or bound constraint in Matrix form.

$$\begin{aligned} \text{Max } Z &= CX \\ \text{Subject to } AX &\leq b \\ X &\geq 0 \\ \text{Where} \end{aligned}$$

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}, \quad x = [x_1, x_2, x_3, \dots, x_n]$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Minimization Problem

For instance, if we are to minimize Z

$$\text{Max } Z = c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n \text{ [Objective function]}$$

$$\begin{aligned} \text{Subject to } &a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n \leq b_1 \\ &a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n \leq b_2 \\ &a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n \leq b_m \end{aligned}$$

$x_i \geq 0, i = 1, 2, 3, \dots, n$ also in Matrix compact form of maximization problem.

$$\text{Min } Z = CX$$

$$\text{Subject to } AX \leq b, \quad x_i \geq 0$$

The only distinguishing difference between the maximization and minimization problems is that of the inequalities of the right hand side of the constraints equation. In maximization problem, the right hand side of the constraints inequalities is " \leq " which that of the minimization is " \geq ".

The maximization problem is the focus of this study

Sensitivity Analysis

Sensitivity analysis (post optimality analysis) after a linear programming problem has been solved it is useful to study the effect of discrete changes in the parameter of the current optimal solution. These may include:

-) Changes in the cost coefficient (C_j)
-) Changes in the right hand side constraints (b_j)
-) Addition of new activity or variable.
-) Changes in existing columns.
-) Addition of new constraints.

In many cases, when these changes occur, it will not be necessary to solve the problem all over again.

Presentation and analysis

Presentation of data

Data are collected in the different sizes of bread namely

14cm x 9cm size (i.e. #100 bread)
15.5cm x 9cm size (i.e. #150 bread)
21cm x 10cm size (i.e. #350 bread)
30cm x 11cm size (i.e. #400 bread)

The table below gives data on; machine hour for cutting, number of labours, oven temperature, quantity of water and bags of flour used on different sizes of bread.

Resources table I

Resources	14cmx9cm size (#100 bread)	15.5cmx9cm size (#150 bread)	21cmx 10cmsize (#350 bread)	30cmx11cm size (#400 bread)
Machine hour for cutting	1 ½	¾	1	2/3
Number of labour	3	2	3	2
Oven (heat) temperature	160 ⁰ C	130 ⁰ C	120 ⁰ C	130 ⁰ C
Quantity of water	110 liters	50 liters	120 liters	60 liters
Bags of flour used	2	2	2	1

Resources table II

Resources	Units
Machine hour for cutting	3
Number of labour	10
Oven (heat) temperature	300 ⁰ C
Quantity of water	340 liters
Bags of flour used	14bags

Profit table

PRODUCT	COST PRICE	SELLING PRICE	PROFIT
A	#60	#72	#12
B	#100	#118	#18
C	#156	#180	#24
D	#252	#280	#28

Formulation of Linear Programming Problem

In formulating linear programming problem, it is necessary to specify the following;

-) Decision variable
-) Objective function
-) Constraints

The decision variable is the variable in which a decision is required to be taken. The decision variables are the keys to a proper and continent formulation of the problem to be clumsy.

In case of product mix decision, the maker has to decide on how may unit of product of A, B, C and D should be produced.

The decision variable will be:

Let the number of unit product A be X_1

Let the number of unit product B be X_2

Let the number of unit product C be X_3

Let the number of unit product D be X_4

Objective function: in case of product mix decision, the decision maker's objective is to maximize the profit in this study, the profits.

The profit per unit of product A is #12

The profit per unit of product B is #18

The profit per unit of product C is #24

The profit per unit of product D is #28

Therefore, the objective function will be:

$$\text{Max } Z = 12x_1 + 18x_2 + 24x_3 + 28x_4$$

Mathematically Equivalents

$$\text{Max } Z = 6x_1 + 9x_2 + 12x_3 + 14x_4$$

Constraints: The constraints are related to machine hours for cutting, numbers of hour, temperature (heat) requirement, quantity of water requirement and number of flour bags used for each size of bread.

In case of this project, product A needs $1\frac{1}{2}$ machine hours for cutting, product B need $\frac{3}{4}$ machine hours, product C needs one machine hour and product D needs $\frac{2}{3}$ machine hours for cutting, the total available machine hours is 3.

$$\rightarrow 1\frac{1}{2} x_1 + \frac{3}{4} x_2 + 1\frac{1}{2} x_3 + \frac{2}{3} x_4 \leq 3$$

Similarly, product A need 3 labours, product B needs 2 labours, product C needs 3 labour and product D needs 2 labours, the total number of the labours available is 10.

The constraint is

$$3X_1 + 2X_2 + 3X_3 + 2X_4 \leq 10$$

Product A needs 160°C , product B needs 130°C , product C needs 120°C and product D needs 130°C and the availability of temperature is 300°C .

The constraint is

$$160X_1 + 130X_2 + 120X_3 + 130X_4 \leq 300$$

Also, product A needs 110 litres of water, product B needs 50 liters of water, product C needs 120 liters of water and product D needs 60 liters of water. The total available water quantity is 340 liters.

The constraint is;

$$110X_1 + 50X_2 + 120X_3 + 60X_4 \leq 340$$

Hence, product A needs 2 bags of flour, product B needs 2 bags of flour, product C needs 2 bags of flour and product D needs a bag of flour. And maximum available of flour bags is 14 bags of flour.

The constraint is

$$2X_1 + 2X_2 + 2X_3 + X_4 \leq 14$$

The combined LPP is

$$\text{Max } Z = 6x_1 + 9x_2 + 12x_3 + 14x_4$$

| Objective function

$$\text{S.t } 1\frac{1}{2} x_1 + \frac{3}{4} x_2 + 1\frac{1}{2} x_3 + \frac{2}{3} x_4 \leq 3$$

| Machine hours

$$3x_1 + 2x_2 + 3x_3 + 2x_4 \leq 10$$

| Number of labours

$$\begin{array}{lcl}
 160x_1 + 130x_2 + 120x_3 + 130x_4 & 300 & | \quad \text{Oven temperature} \\
 110x_1 + 50x_2 + 120x_3 + 60x_4 & 340 & | \quad \text{Quantity water} \\
 2x_1 + 2x_2 + 2x_3 + 1x_4 & 14 & | \quad \text{Number of flour bags} \\
 x_1, x_2, x_3, x_4 & = & | 0
 \end{array}$$

The LPP above is solved by converting it to the following standard form

$$\begin{array}{l}
 \text{Max } Z = 6x_1 + 9x_2 + 12x_3 + 14x_4 \\
 \text{s.t.} \quad 18x_1 + 9x_2 + 12x_3 + 8x_4 + x_5 + 0x_6 + 0x_7 + 0x_8 + 0x_9 = 36 \\
 3x_1 + 2x_2 + 3x_3 + 2x_4 + 0x_5 + x_6 + 0x_7 + 0x_8 + 0x_9 = 10 \\
 160x_1 + 130x_2 + 120x_3 + 130x_4 + 0x_5 + 0x_6 + x_7 + 0x_8 + 0x_9 = 300 \\
 110x_1 + 50x_2 + 120x_3 + 60x_4 + x_5 + 0x_6 + 0x_7 + x_8 + 0x_9 = 340 \\
 2x_1 + 2x_2 + 2x_3 + 1x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8 + x_9 = 14 \\
 x_1, x_2, x_3, \dots, x_9 \geq 0
 \end{array}$$

- Linear Solution (Initial Table)

BV	CB	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	XB	MR
X ₅	0	18	9	12	8	1	0	0	0	0	36	36/8 = 4.5
X ₆	0	3	2	3	2	0	1	0	0	0	10	5
X ₇	0	160	130	120	130	0	0	1	0	0	300	2.31
X ₈	0	110	50	120	60	0	0	0	1	0	340	5.67
X ₉	0	2	2	2	1	0	0	0	0	1	14	14
	C _j	6	9	12	14	0	0	0	0	0		
C _b – P _j	Z _j	0	0	0	0	0	0	0	0	0		
C _j - Z _j	ζ _j	6	9	12	14	0	0	0	0	0		

The solution is not optimal because C_j – Z_j row has positive values.

X₄ is incoming vector because X₄ has the most positive value and X₇ is the outgoing vector while 130 is the pivot element from the maximum ratio column since X₇ has the smallest value.

Changing the Objectives Function Co-Efficient Of A Basic Variable

Suppose we want to determine the coefficient of changes on the unit price of product D (C₄). It is

clear that when C₁ decreases below a certain level. It may not be profitable to include product D in the optimal product mix. On the other hand, if C₄ increase, it is possible that it may change the optimal product mix at some level because product D may become so profitable that the optimal product mix may include product E, hence there is an upper and lower limit on the variable of C₄ within which the optimal solution is not affected.

Optimal Table

BV	CB	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	XB
X ₅	0	2	-7	-4	0	1	0	0	0	0	-76
X ₆	0	-1	-1	-1	0	0	1	0	0	0	-18
X ₄	13	16/13	12/13	12/13	1	0	0	1/130	0	0	
X ₈	0	-10	-70	0	0	0	0	0	1	-60	-500
X ₉	0	10/13	1	14/13	0	0	0	-1/13	0	1	122/13
	C _j	6	9	12	13	0	0	0	0	0	
C _b - P _j	Z _j	208/13	12	12	13	0	0	13/130	0	0	290/13
C _j - Z _j	ζ _i	-10	-4	0	0	0	0	-13/130	0	0	

The solution is optimal and it is given as $X_4 = 13$ and the maximum profit = **N30**

When the value of C_4 goes below the range provided by sensitivity analysis the optimal table will no longer be optimal as one of the non-basic $Z_j - C_j$ will become negative which was illustrated above.

Changing the Price of Both Non Basic Variables

In this case, the profit of all the flour products can be changed in such a way that the objective functions becomes $Z = 7x_1 + 10x_2 + 11x_3 + 16x_4$, the effect on the optimal product mix can be determined by changing $Z_j - C_j$ row in the optimal table remain positive.

$$\begin{aligned}
 [Z_1 - C_1] &= [0 \ 0 \ 16 \ 0 \ 0] \begin{bmatrix} 2 \\ -1 \\ 16/13 \\ -10 \\ 10/13 \end{bmatrix} - 7 = \frac{256}{13} - 7 \geq \frac{165}{13} \\
 [Z_2 - C_2] &= [0 \ 0 \ 16 \ 0 \ 0] \begin{bmatrix} -7 \\ -2 \\ 1 \\ -70 \\ 1 \end{bmatrix} - 10 \rightarrow 16 - 10 \rightarrow 6 \geq 10 \\
 [Z_3 - C_3] &= [0 \ 0 \ 16 \ 0 \ 0] \begin{bmatrix} 4 \\ -1 \\ 12/13 \\ -70 \\ 1 \end{bmatrix} - 11 \rightarrow \frac{92}{13} - 11 \geq 0 \\
 \frac{192 - 143}{13} &\geq 0 \rightarrow \frac{49}{13} \geq 0 \\
 [Z_4 - C_4] &= [0 \ 0 \ 16 \ 0 \ 0] \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} - 16 \rightarrow 16 - 16 \geq 0 \\
 Z_5 - C_5 &\geq 0, Z_6 - C_6 \geq 0 \\
 [Z_7 - C_7] &= [0 \ 0 \ 16 \ 0 \ 0] \begin{bmatrix} 0 \\ 0 \\ 1/130 \\ 0 \\ -1/13 \end{bmatrix} - 0 \rightarrow \frac{16}{130} - 0 \geq 0
 \end{aligned}$$

$$[Z_8 - C_8] = [0 \ 0 \ 16 \ 0 \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - 0 \rightarrow 0 - 0 \geq 0$$

$$[Z_9 - C_9] \geq 0$$

In order to study the effect of changes in b_i 's, it is sufficient to verify whether the new b_i 's remain positive or not, to do this, we know that only column in the table including vector b_i 's can be obtained by multiplying the corresponding

column in the initial tableau by the inverse, by the basis column in the case, the basic column are variable $[X_5 + X_6 + X_7 + X_8 + X_9]$ in the initial column tableau.

Hence, the basic matrix corresponding to tableau 1 is given by the

$$B^{-1} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/130 & 0 & 0 \\ 0 & 0 & 0 & 1 & -60 \\ 0 & 0 & -1/13 & 0 & 1 \end{bmatrix}$$

The inverse of basis matrix can be gotten by using reversed simplex method. The inverse of matrix above is the column corresponding to $[X_5 + X_6 + X_7 + X_8 + X_9]$ in 1.

The initial table given the inverse of basic matrix B is

$$B^{-1} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/130 & 0 & 0 \\ 0 & 0 & 0 & 1 & -60 \\ 0 & 0 & -1/13 & 0 & 1 \end{bmatrix}$$

The inverse of the new right-hand side constraints b_i 's in the initial table due to an increased hour given by

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/130 & 0 & 0 \\ 0 & 0 & 0 & 1 & -60 \\ 0 & 0 & -1/13 & 0 & 1 \end{bmatrix} \begin{bmatrix} 37 \\ 10 \\ 300 \\ 340 \\ 14 \end{bmatrix} = \begin{bmatrix} 37 \\ 10 \\ 300/130 \\ 340 - 840 \\ -300/13 + 14 \end{bmatrix} = \begin{bmatrix} 37 \\ 10 \\ 30/13 \\ -500 \\ 118/13 \end{bmatrix}$$

The new optimal product mix is $X_1 = X_2 = X_3 = 0$ and $X_4 = 30/13$

The new maximum value of $Z = 6X_1 + 9X_2 + 12X_3 + 14X_4$

$$Z = 6(0) + 9(0) + 12(0) + 14(30/13) = \text{N}32.31\text{k}$$

Since the profit of product D is equal to the profit of additional one unit to the machine hours which means there is no effect.

Suppose on addition one unit of labour is also made available and bakery interested in determine

how this affects the optimal product mix. The addition of one unit labour changes the vector of the right-hand side constraint b_i in the initial simplex tableau form.

$$\begin{bmatrix} 36 \\ 11 \\ 300 \\ 340 \\ 14 \end{bmatrix} \text{ to } \begin{bmatrix} 36 \\ 11 \\ 300 \\ 340 \\ 14 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/130 & 0 & 0 \\ 0 & 0 & 0 & 1 & -60 \\ 0 & 0 & -1/13 & 0 & 1 \end{bmatrix} \begin{bmatrix} 36 \\ 11 \\ 300 \\ 340 \\ 14 \end{bmatrix} = \begin{bmatrix} 36 \\ 11 \\ 300/130 \\ 340 - 840 \\ -300/13 + 14 \end{bmatrix} = \begin{bmatrix} 36 \\ 11 \\ 30/13 \\ -500 \\ 118/13 \end{bmatrix}$$

The product mix still remains $X_4 = \frac{30}{13}$, while $X_1 = X_2 = X_3 = 0$ and $Max Z = \$32.31k$ with this result is advisable to increase the number of labour since total profit increase to \$32.31k.

Likewise, if an additional one bag of flour is added. The addition of one bag of flour changes the vector of right-hand side constraints b_i in the initial simplex tableau form.

$$\begin{bmatrix} 36 \\ 11 \\ 300 \\ 340 \\ 14 \end{bmatrix} \text{ to } \begin{bmatrix} 36 \\ 11 \\ 300 \\ 340 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/130 & 0 & 0 \\ 0 & 0 & 0 & 1 & -60 \\ 0 & 0 & -1/13 & 0 & 1 \end{bmatrix} \begin{bmatrix} 36 \\ 11 \\ 300 \\ 340 \\ 15 \end{bmatrix} = \begin{bmatrix} 36 \\ 11 \\ 300/130 \\ 340 - 900 \\ -300/13 + 15 \end{bmatrix} = \begin{bmatrix} 36 \\ 11 \\ 30/13 \\ -560 \\ 108/13 \end{bmatrix}$$

The product mix still remains $X_4 = \frac{30}{13}$, while $X_1 = X_2 = X_3 = 0$ and $Max Z = \$32.31k$ with this result is advisable to increase the number of bag of flour since total profit increase to \$32.31k.

Suppose a new product E [X_{10}] requires 2 machine hours for cutting 6 labour 140 liters of water 180°C of temperature and 6 bags of flour, with a profit \$20. The bakery wants to know whether it is economical to include E [X_{10}]

Adding a New Activity

The column X_{10} will be

$$\begin{bmatrix} 2 \\ 6 \\ 140 \\ 180 \\ 6 \end{bmatrix}$$

The initial table, the optimal table mix given by second table will be optimal as long as the relative profit coefficient $Z_j - C_j$ of the new product namely $Z_{10} - C_{10}$ is positive.

From the revised simplex method we know that

$$\pi = C_B B^{-1} C_{10} = 20$$

Where

$$P_{10} = \begin{bmatrix} 2 \\ 6 \\ 140 \\ 180 \\ 6 \end{bmatrix}$$

$$\bar{C}_{10} = [0 \ 0 \ 14 \ 0 \ 0] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/130 & 0 & 0 \\ 0 & 0 & 0 & 1 & -60 \\ 0 & 0 & -1/13 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 140 \\ 180 \\ 6 \end{bmatrix} - 20 = [0 \ 0 \ 14 \ 0 \ 0] = \begin{bmatrix} 2 \\ 6 \\ 14/13 \\ 180 - 360 \\ -140/13 + 6 \end{bmatrix} - 20$$

$$\bar{C}_{10} = [0 \ 0 \ 14 \ 0 \ 0] = \begin{bmatrix} 2 \\ 6 \\ 14/13 \\ -180 \\ -62/13 \end{bmatrix} - 20 = \frac{196}{13} - 20$$

$$\bar{C}_{10} = -\frac{64}{13}$$

Since $Z_{10} - C_{10} \leq 0$, it turns out the new activity can contribute to an increase profit.

Next Iteration All Iterations Write to Printer

Iteration 1										
Basic	x1	x2	x3	x4	sx5	sx6	sx7	sx8	sx9	Solution
z (max)	-6.00	-9.00	-12.00	-14.00	0.00	0.00	0.00	0.00	0.00	0.00
sx5	18.00	9.00	12.00	8.00	1.00	0.00	0.00	0.00	0.00	36.00
sx6	3.00	2.00	3.00	2.00	0.00	1.00	0.00	0.00	0.00	10.00
sx7	160.00	130.00	120.00	130.00	0.00	0.00	1.00	0.00	0.00	300.00
sx8	110.00	50.00	120.00	60.00	0.00	0.00	0.00	1.00	0.00	340.00
sx9	2.00	2.00	2.00	1.00	0.00	0.00	0.00	0.00	1.00	14.00
Lower Bound	0.00	0.00	0.00	0.00						
Upper Bound	infinity	infinity	infinity	infinity						
Unrestr'd (y/n)?	n	n	n	n						
Iteration 2										
Basic	x1	x2	x3	x4	sx5	sx6	sx7	sx8	sx9	Solution
z (max)	11.23	5.00	0.92	0.00	0.00	0.00	0.11	0.00	0.00	32.31
sx5	8.15	1.00	4.62	0.00	1.00	0.00	-0.06	0.00	0.00	17.54
sx6	0.54	0.00	1.15	0.00	0.00	1.00	-0.02	0.00	0.00	5.38
x4	1.23	1.00	0.92	1.00	0.00	0.00	0.01	0.00	0.00	2.31
sx8	36.15	-10.00	64.62	0.00	0.00	0.00	-0.46	1.00	0.00	201.54
sx9	0.77	1.00	1.08	0.00	0.00	0.00	-0.01	0.00	1.00	11.69
Lower Bound	0.00	0.00	0.00	0.00						
Upper Bound	infinity	infinity	infinity	infinity						
Unrestr'd (y/n)?	n	n	n	n						

Results and Discussion

At the optimality stage, as long as product A is less than #17.23k, is not economical to produce product A if C_j is increases to #18.00k, the new optimal product is to produce (30/16) i.e. two units of product A to make a total profit of #33.75k. Also, as long as the unit profit of B is less than #14, it is not economical to produced product 13, if C_2 is increases to #15, the new optimal product mix to produce (30/13) i.e. two unit of product B with a total profit of #34.62K.

It is clear that when C_4 decreases below a certain level, it may not be profitable to include product D in the optimal product. Assuming C_4 is greater than or equal to 13 the new maximum profit, is #30.00k.

Conclusion

In conclusion, when the price of both basic and non-basic product changed, the objective function also change from $\text{Max } Z = 6x_1 + 9x_2 + 12x_3 + 14x_4$ to $\text{Max } Z = 7x_1 + 10x_2 + 11x_3 + 16x_4$. Hence the total profits increase to #36.92k.

Suppose an additional one hour is added to the machine hours, the new optima product mix is $x_1 = x_2 = x_3 = 0$ and $x_4 = 30/13$ and the total profit is #32.31 k with this, it is advisable to increase the machine hours because the profit increase. Also with an additional one unit of security of of labour is made available and the available product mix still remain as $x_4 = 30/13$ while with a profit of #32.31k. like wise when an additional one bag of flour is added, the rprofit mix is then same profit is increase to #32.31 k. Hence, it is advisable to increases to increases the hour. labour and number of bags of flour used.

Based on the conclusion, the following recommendation are made:

- The bakery should increase the selling the price of products (A C: D) in order to make it profitable for the company in the face of existing cost of production;
- The management of the bakery should apply linear programming (and sensitivity

analysis) to make optimal use of time activities and employees' services in order to maximize profit.

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