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THE PERFORMANCE OF CONTROL CHARTS AND CUSUMS UNDER LINEAR TREND WITH WEIBULL DISTRIBUTION

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Abstract

A different approach in which the operation of the scheme is regarded as forming a Markovian chain is set out. The run length properties of control charts and cusum charts, under conditions of slippage in mean level by step change to a new sustained level are well documented. Such control procedures are often used where a genuine out of control signal may result from gradual, rather than step changes. The paper presents the results of evaluation of run length under linear trend with weibull distribution

1. INTRODUCTION

CUSUM schemes were introduced in 1954 by 'E.S.Page' [1]. These charts may be used in several situations where a production process is expected to change at an unknown time from an in control state to an out of control state. As soon as one has evidence that the out of control state has observed one would wishes to stop the production process to take remedial measures. CUSUM schemes have proven to be optimal stopping rules in the sense that to minimise expected run length under the out of control state given that the stopping rules has a fixed expected run length under the in-control state .

CUSUM control charts have found an interesting variety of applications since their introduction. Several researchers namely Johnson[3], U.Rendtel [2], James M.Lucas and Ronald B. Crosier [4], Brook and Evans [5], Sullivan,J.H. and Woodall,W.H.[8], Hawkins, D.M. [9], Hinkley, D.V. [10], Sen, Ashish, and Srivastava, Muni S [11], Khan R.A., [12] have attempted performance of CUSUM charts under various conditions. In most of the research problems the CUSUM chart performance is mainly assessed based on the ARL or its distribution. In other words the effectiveness of monitoring procedures like Shewart charts with Action limit only, control charts with Warning lines and CUSUM procedures can be demonstrated when there is a slippage in mean level from a target value. This can be done with the help of ARL or other features of run length distributions. The ARL is usually measured on the assumption of step change i.e. abrupt change from the process average. The main purpose of this chapter is to assess the performance of control charts and CUSUM charts under linear trend with non normal distributions namely, Exponential distribution, Gamma distribution and Weibull distribution. These distributions are considered because of their heavy applications in the real world. These distributions importance is discussed as and when the CUSUM schemes and other control charts performance is assessed. In the subsequent section we discuss Shewart chart with action line, control chart with warning line and CUSUM procedures.

2. SHEWART CONTROL CHART WITH ACTION LINES

In the construction of control charts we are using two sets of limits such as action limits or outer limits and warning limits or inner limits. When a in action lines point plots outside of this limit, a search for an assignable causes is made and corrective action is taken if necessary. Shewart control chart with only action lines, it is denoted by 'Scheme A' and specified distributional assumptions, the evaluation of run length properties follows Geometric distribution with parameter P_A that is

$$\text{ARL} = \frac{1}{P_A} \qquad (1)$$

where P_A is the Probability of action limit for a specified process mean and distribution form

3. SHEWART CONTROL CHART WITH WARNING LINES

In a Shewart control chart with action and warning lines we take decisions with monitoring procedures depend on preceding observations as well as the most recent value. It is denoted by 'Scheme W'. If one or more points fall between the warning line and the central line or very close to the warning line, we should be suspicious that the process may not be operating properly. One possible action to take when this occurs is to increase the sampling frequency. The use of warning limit can increase the sensitivity of the control chart. These decisions can be taken with the help of Markov chains [Page, 1954]. The complete run length distribution is obtained by using successive powers of the transition matrix. In particular, the ARL is found to be

$$\frac{1+P_W-P_A}{P_A+P_W(P_W-P_A)} \qquad (2)$$

where P_W is the probability of a violation of warning line which includes more extreme action line violation P_A. The action line scheme having only two states one Transient, one absorbing.

4. TRANSITION MATRICES FOR CONTROL CHART AND CUSUMSWITH WEIBULL DISTRIBUTION

4.1. THE IMPORTANCE OF WEIBULL DISTRIBUTION

Parametric distributions are often used to model life time and time-to-failure responses. The Weibull distribution, a member of a special class of parametric distributions known as location-scale distributions, has found wide application in engineering and medical research. The Weibull distribution is characterized by its shape and scale parameters. By changing the shape parameter, the Weibull distribution can be made to have many different shapes, from highly skewed like an exponential distribution to nearly bell-shaped like a normal distribution. The hazard function, an important characteristic of a life time distribution, indicates the instantaneous failure rate of surviving units. The Weibull is unique in that its hazard function can model increasing, decreasing, or constant hazard rates. The practical importance of the Weibull distribution stems from its ability to model life time phenomena with many different commonly occurring shapes and hazard rates. This talk will first describe various important mathematical quantities relevant to life time modeling, including the probability density function, the hazard function and percentiles. Relating life time responses to covariates, like drug type and age in a medical study or temperature in an industrial experiment, will then be discussed. Finally, example applications of the Weibull distribution in engineering and medical research will be presented. Weibull distribution is often used to represent observed values in 'life testing' types of situations. The successive observations are represented by independent random variables X_1, X_2-------X_n having the Weibull probability density function

$$P(X_i/\theta) = \theta^{-1}\, C\, X_i^{c-1} e^{-(x_i^{c}/\theta)}$$
$$(x_i > 0,\ \theta > 0,\ C > 0) \qquad (3)$$

$$P(x_i \le X) = e^{-(-x_i^{c}/\theta)} \quad (X > 0) \qquad (4)$$

The mean of the distribution is

$$\xi = \theta^{c-1}\, \Gamma\,(C^{-1} + 1) \qquad (5)$$

Generally in life testing problems, it has been found that a Weibull distribution [for which the c^{th} power of the variable is Exponential distributed] often gives the markedly more accurate representation. If C = 1, above density function reduces to Exponential distribution.

Certain results are obtained using Markovian approach as discussed in the following situation.

Weibull distribution has been extensively used in life testing, Reliability and Quality control problems. Weibull [14] showed that the distribution is also useful in describing the 'Wear – out 'or fatigue failures. Kao [15] used it as a model for vacuum tube failures while Lieblein and Zelen [16] used it as a model for ball bearing failures. Mann [17]gives a variety of situations in which the distribution is used for other types of failures data. Some indication of the recent popularity of the Weibull distribution is seen in split stone'sthesis. The distribution is often suitable where the conditions of 'Strictrandomness' of the exponential distribution are not satisfied.

Berrettoni [18] has described many applications of the Weibull distribution, using graphical methods in most cases. Sometimes the Exponential distribution will be found to suffice, and the Weibull distribution will be dispensed. However, the Weibull distribution may provide just the extra flexibility needed to make a model sufficiently accurate for use is an analysis. The Weibull distribution is sometimes used as a tolerance distribution in the analysis of quantal response data. The explicit form of its cumulative distribution function makes it especially suitable for this purpose.

4.2. TRANSITION MATRIX FOR CONTROL CHART

The Transition matrices representation for control charts are give below. In case of row labels refer to states at sample (i – 1) and column heading to states at sample i. The upper left column partition is the reduced transition matrix after deleting row and column for the absorbing states.

Table 1

A. Transition matrix for "Action only" (Shewart) chart

	clear	signal
clear	$1\text{-}P_A$	P_A
signal	0	1

Table 2

W. Transition matrix for "Action and Warning" control chart

	clear	Warning	signal
clear	$1\text{-}P_A$	$P_W\text{-}P_A$	P_A
Warning	$1\text{-}P_W$	0	P_W
Signal	0	0	1

4.3.TRANSITION MATRIX FOR CUSUM CHART WITH WEIBULL DISTRIBTUION

Brooks and Evans [5] show that CUSUM procedures may be viewed as Markov chains. However for, continuous distributions, it is necessary to consider the discretization for the markov chain representation and the various states then corresponds to values of the CUSUM at any step. For an instance consider a scheme with decision interval H and reference value K are designed to detect upward shift from a target value. A set of (m + 1) states can be interpreted as the CUSUM values of

$$\leq 0,\ 0 \text{ to } \frac{H}{m},\ \frac{H}{m} \text{ to } \frac{2H}{m},\ \text{etc--------------}(m-1)\frac{H}{m} \text{ to } < H,\ \text{----------} \geq H. \qquad (6)$$

The last of these states that is violation of the decision interval can be thought of as an absorbing barrier.

In the usual Markov chain notation with Transition matrix P and reduced matrix are which is obtained from the deletion of the row and column representing the absorbing barrier. The well known result for obtaining ARL from an initial zero CUSUM is the sum of the elements in the first row of $(I - R)^{-1}$. While considering the states the degree of discretization has some effect on the accuracy of ARL determination. In the present study 20 states transition matrices were used. Thus for H = 5 and K = 0.5 with μ at the target value. The transition matrix in general and for the particular study is shown in tables 3 and 4 respectively.

Table 3

C.Transition matrix for CUSUM scheme H, K (m + 1 states)

		0	$\frac{H}{m}$	$\frac{2H}{m}$	- -------------	$\frac{(m-1)H}{m}$	≥H
CUSUM at (i - 1)th sample	≤0	$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	--------------	$P_{0,m-1}$	$P_{0,m}$
	$\frac{H}{m}$	$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	--------------	$P_{1,m-1}$	$P_{1,m}$
	$\frac{2H}{m}$	$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	---------------	$P_{2,m-1}$	$P_{2,m}$
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
	$\frac{(m-1)H}{m}$	$P_{m-1,0}$	$P_{m-1,1}$	$P_{m-1,2}$	---------------	$P_{m-1,m-1}$	$P_{m-1,m}$
	≥H	0	0	0	--------------	0	1

The entries in the above matrix need some explanation. In the first row, all entries corresponding moves from an initial zero CUSUM, and in the first entry, it indicates that a sample i, the CUSUM remains at or below zero. This means the sample value should not exceed the reference value k. Thus

$$P_{0,0} = P(x \leq K) \qquad (7)$$

For a move from state zero to H/m, the ith sample must have a value between K and

(K +H/m). So that the subtraction of reference value gives a CUSUM contribution of H/m. After the discretization,

$$P_{0,1} = P(x = K + 2H/m) \qquad (8)$$

For details including discretization, see Brook and Evans [5]

5.RUN LENGTH CALCULATION UNDER LINEAR TREND WITH WEIBULL DISTRIBUTION

The method explained for ARL calculations is applicable only under the assumption that the phenomenon or process average undergoes stable distribution.

However, in case of liner trend both the distribution functions and transition matrix changes over the time. These changes can be quantifiable for any specific rate of slippage. Here we get a non-homogenous Markov chain and an alternative method of obtaining ARL is essential. This can be easily derived from the procedures for generalising rum length distribution under stable conditions.

The entry $P_{0,m}$in the transition matrix stands for the probability of occurrence of a signal at the first sample instant. That means $P^{i}_{0,m}$ stands for element in column 0, row m ofP^{i}. This givesprobability of signal atthe ith sample. Successive difference between say $P^{i}_{0,m}$ -$P^{i-1}_{0,m}$ gives the probability of signal at the ith sample. For an increasing i, we get probability distribution of run length.

For an illustration, consider W chart with action line at $3.09\sigma_e$ from a target value and warning limit at $1.96\sigma_e$. Let the process changes to a value $1.00\sigma_e$ from the target value, then for Weibull variable

$$P_A = 1- \phi\ (3.09\text{-}1) = 0.014158$$

$$P_W = 1- \phi\ (1.96\ \text{-}1) = 0.142943$$

The transition matrix in this case is given by

$$P = \begin{bmatrix} 0.8570570 & 0.128784 & 0.014157 \\ 0.8570570 & 0 & 0.142943 \\ 0 & 0 & 1 \end{bmatrix}$$

The ARL is

$$\frac{1+P_W-P_A}{P_A+P_W(P_W-P_A)} = 34.660575$$

The probability of a signal at the first sample $P_{0,3}$ is 0.014157. Squaring P, we get

$$\begin{bmatrix} 0.8449229 & 0.110376 & 0.0447009 \\ 0.7345468 & 0.110376 & 0.1550770 \\ 0 & 0 & 1 \end{bmatrix}$$

The element $P^{2}_{0,3}$ is 0.0447009. Obviously the probability of a signal a sample 2 is 0.0305439. Similarly $P^{3}_{0,3}$ is 0.0724408, gives 0.02773983is the probability run length for three samples.

In the case of non homogenous transition matrix, it is necessary to multiply original P by new transition matrix obtained after allowing a step change in the mean level.

We denote the rate of change by Δ and we use ^{1}P, ^{2}P etc, for the first, second etc samples transition matrices are deduced. In general the Cumulative probability of signal at or before the ith sample is (0, m)th element of the product.

$$\text{i.e.}\ {}^{1}P\ {}^{2}P\ {}^{3}P\text{-------}{}^{i}P$$

Individual terms of the run length probability distribution are obtained by successive differences of cumulative probabilities.

Reconsider W scheme with A = 3.09, W = 1.96, with 0 shift, we get

$$P_A = 0.0002004$$
$$P_W = 0.0204326$$

We get

$$^{1}P = \begin{bmatrix} 0.97956732 & 0.02023223 & 0.00020044 \\ 0.97956732 & 0 & 0.02043267 \\ 0 & 0 & 1 \end{bmatrix}$$

At the second sample, the $0.5\sigma_e$ shift i.e.Δ = 0.5 gives

$$P_A = 0.002427272$$
$$P_W = 0.063858624$$

We get

$$^{2}P = \begin{bmatrix} 0.93614137 & 0.0614313 & 0.00242727 \\ 0.93614137 & 0 & 0.06385862 \\ 0 & 0 & 1 \end{bmatrix}$$

Also

$$^{1}P\ ^{2}P = \begin{bmatrix} 0.93595372 & 0.06017614 & 0.0038701 \\ 0.91701350 & 0.06017614 & 0.0228103 \\ 0 & 0 & 1 \end{bmatrix}$$

From the above calculations we get the cumulative probabilities of a signal at sample 2 is 0.0038701. Subtracting the (0, m)th element of ^{1}P gives 0.00366966. The above computations are presented for the sake of illustration allowing slippages at different levels similar type of calculated and re-designated as

$$^{1}P\ ^{2}P\ ^{3}P\text{-------}$$

For different slippages in mean level, say Δ = 0.01, 0.02, 0.05, 0.1, 0.2 are computed and summarised below.

TABLE 4

SIGNALING TABLE FOR ACTION AND WARNING CHARTS

											SIGNAL
SHIFT	**P_A**	**P_W**		0.979567322	0.020232231	0.000200447					0.001120045
0	0.000200447	0.02	**^{1}P**	0.979567322	0	0.020432678					
				0	0	1		0.97881452	0.020352128	0.000833352	
							^{1}P *^{2}P	0.959006947	0.020352128	0.020640925	0.009329056
0.01				0.979010759	0.020776651	0.00021259		0	0	1	
	0.00021259	0.021	**^{2}P**	0.979010759	0	0.020989241					
				0	0	1		0.978235031	0.02088385	0.000881119	
							^{2}P*^{3}P	0.957906262	0.02088385	0.021209888	0.016852852
0.02				0.978443039	0.021331584	0.000225378		0	0	1	
	0.000225378	0.022	**^{3}P**	0.978443039	0	0.021556961					
				0	0	1		0.976451649	0.022563225	0.000985127	
							^{3}P*^{4}P	0.955617693	0.022563225	0.021819083	0.025974939
0.05				0.976671768	0.023060335	0.000267896		0	0	1	
	0.000267896	0.023	**^{4}P**	0.976671768	0	0.023328232					
				0	0	1		0.973225054	0.025549373	0.001225573	
							^{4}P*^{5}P	0.950776144	0.025549373	0.023674483	0.055767695
0.1				0.973485847	0.026159631	0.000354522		0	0	1	
	0.000354522	0.027	**^{5}P**	0.973485847	0	0.026514153					
				0	0	1		0.965846876	0.032326596	0.001826529	
							^{5}P*^{6}P	0.940571718	0.032326596	0.027101687	0.077200649
0.2				0.966189411	0.033207053	0.000603536		0	0	1	
	0.000603536	0.034	**^{6}P**	0.966189411	0	0.033810589					
				0	0	1		0.935576381	0.059354321	0.005069298	
							^{6}P*^{7}P	0.904489885	0.059354321	0.036155794	0.160353622
0.5				0.936141376	0.061431351	0.002427273		0	0	1	
	0.002427273	0.064	**^{7}P**	0.936141376	0	0.063858624					
				0	0	1		0.85497677	0.120560962	0.024462269	
							^{7}P*^{8}P	0.802326595	0.120560962	0.077112444	0.220349962
1				0.857057081	0.128784994	0.014157925		0	0	1	
	0.014157925	0.143	**^{8}P**	0.857057081	0	0.142942919					
				0	0	1					

In case of CUSUM scheme the transition matrix can be obtained by making use of the formulae given in section 4.3.

6.RUN LENGTH PROPERTIES OF STANDARD CONTROL CHARTS UNDER LINEAR TREND WITH WEIBULL DISTRIBUTION

The following table give the average run length and other properties of the run length distributions for three basic control charts namely A scheme, W scheme and CUSUM scheme. In these tables Δ values ranges from 0 to 1. Here we note that the shift will be detected rapidly when the trend is greater than 1 standard error per sample in all charts methods.

TABLE 5

Run Length Properties of Control Procedures Under Linear Trend				
		SCHEME		
Δ		**A**	**W**	**C**
0	ARL	450.6919	115.8865	31.21505
	Δ*ARL	0	0	0
0.01	ARL	426.5498	112.6495	31.23813
	Δ*ARL	4.265498	1.126495	0.312381
0.02	ARL	403.9013	109.5046	31.26118
	Δ*ARL	8.078025	2.190092	0.625224
0.05	ARL	343.9257	100.5977	31.33018
	Δ*ARL	17.19629	5.029887	1.566509
0.1	ARL	265.61	87.37712	31.44459
	Δ*ARL	26.561	8.737712	3.144459
0.2	ARL	163.8623	66.117	31.67124
	Δ*ARL	32.77245	13.2234	6.334247
0.5	ARL	48.74612	29.99123	32.33215
	Δ*ARL	24.37306	14.99561	16.16607
1	ARL	11.84612	10.19315	33.35901
	Δ*ARL	11.84612	10.19315	33.35901

The entries in the third and fourth column are obtained by using equation (1) and (2) the first column ARL are obtained by using from initial zero CUSUM is the sum of the elements in the first row of $(I-R)^{-1}$ of respective slippage values. Here CUSUM scheme is operated with H = 5, K = 0.5 under Weibull distribution. The following table gives values of ARL and Δ*ARL (displacement ARL) for these basic control charts and the corresponding data for further two CUSUM scheme with H =8, K=0.25 and H = 2.5, K =1.

Table 6
Further Run Length Data for Control Schemes

	CONTROL CHARTS								CUSUM SCHEMES					
	A=3.09		A=3		A=3.09,W=1.96		A=3,W=2		H=8,K=0.25		H=5,K=0.5		H=2.5,K=1	
Δ	ARL	Δ*ARL	ARL	Δ*ARL	ARL	Δ*ARL	ARL	Δ*ARL	ARL	Δ*ARL	ARL	Δ*ARL	ARL	Δ*ARL
0	450.69	0	317.348329	0	115.89	0	120.915063	0	238.529554	0	259.377919	0	249.531165	0
0.005	438.43	2.19213721	309.463949	1.54731975	114.26	0.57128184	119.018182	0.59509091	138.944839	0.6947242	131.226592	0.65613296	120.040585	0.60020293
0.01	426.55	4.26549844	301.808616	3.01808616	112.65	1.12649549	117.153482	1.17153482	77.5294484	0.77529448	81.2262769	0.81226277	81.0397792	0.81039779
0.015	415.05	6.22568234	294.37486	4.41562291	111.07	1.66598596	115.32042	1.7298063	38.5293442	0.57794016	71.2259651	1.06838948	70.0389783	1.05058468
0.02	403.9	8.07802542	287.155481	5.74310963	109.5	2.1900921	113.518461	2.27036922	32.5413772	0.65082754	61.2256564	1.22451313	60.0381827	1.20076365
0.025	393.1	9.82761463	280.143534	7.00358835	107.97	2.69914676	111.747078	2.79367695	28.52914	0.7132285	43.225351	1.08063377	50.0373923	1.25093481
0.03	382.64	11.4792994	273.332322	8.19996965	106.45	3.19347685	110.005754	3.30017261	22.5888834	0.6776665	37.2831033	1.1184931	42.9756484	1.28926945
0.04	362.68	14.5072327	260.286492	10.4114597	103.48	4.13924179	106.611248	4.26444994	20.6157341	0.82462936	31.3072075	1.2522883	39.9455852	1.59782341
0.05	343.93	17.1962874	247.969007	12.3984503	100.6	5.0298869	103.330963	5.16654814	38.6395701	1.9319785	28.3301769	1.41650884	35.9185224	1.79592612
0.06	326.3	19.5777961	236.334265	14.1800559	97.797	5.86782285	100.16104	6.00966242	18.6630612	1.11978367	24.3531177	1.46118706	31.8914949	1.91348969
0.08	294.12	23.5295738	214.946039	17.1956831	92.435	7.39477969	94.1374694	7.53099755	15.7090194	1.25672155	21.3989136	1.71191309	29.8375447	2.38700357
0.1	265.61	26.5609974	195.815587	19.5815587	87.377	8.737712	88.5120724	8.85120724	12.5421303	1.25421303	18.2381295	1.82381295	27.0269869	2.70269869
0.15	207.49	31.1229153	156.202959	23.4304439	75.961	11.3942094	76.0197414	11.4029612	10.8594122	1.62891183	16.558286	2.4837429	25.6498001	3.84747001
0.2	163.86	32.7724506	125.815829	25.1631659	66.117	13.2233999	65.4817406	13.0963481	8.95727525	1.79145505	14.6712354	2.93424708	22.5167009	4.50334018
0.25	130.77	32.692202	102.283431	25.5708577	57.637	14.4091652	56.5832257	14.1458064	7.04759448	1.76189862	11.7834233	2.94585583	19.3844143	4.84610358
0.3	105.41	31.6217093	83.8932307	25.1679692	50.337	15.1010534	49.0587712	14.7176314	6.1307555	1.83922665	10.8948286	3.26844857	15.2529206	4.57587618
0.4	70.442	28.176648	57.8725064	23.1490026	38.651	15.4603998	37.27591	14.910364	5.2771782	2.11087128	9.11519804	3.64607922	12.9922399	5.19689597
0.5	48.746	24.3730583	41.184584	20.592292	29.991	14.9956146	28.7498772	14.3749386	4.39971112	2.19985556	7.3321493	3.66607465	10.7345292	5.36726462
0.6	34.825	20.8947362	30.1565076	18.0939045	23.551	14.1303383	22.5161232	13.5096739	4.01477953	2.40886772	6.54548348	3.92729009	8.47967969	5.08780781
0.8	19.346	15.4770006	17.448592	13.9588736	15.094	12.0754473	14.4513443	11.5610754	3.6544577	2.92356616	5.96054392	4.76843514	7.97822347	6.38257878
1	11.846	11.8461154	11.0231764	11.0231764	10.193	10.1931469	9.81914279	9.81914279	3.57216861	3.57216861	5.35901488	5.35901488	5.48735686	5.48735686
1.25	7.1811	8.97637564	6.87731425	8.59664282	6.6843	8.35539024	6.50364139	8.12955174	3.39788804	4.24736004	4.83212906	6.04016132	4.88825044	6.11031304
1.5	4.8175	7.22628179	4.70767115	7.06150673	4.6891	7.03362348	4.60873849	6.91310774	2.89151919	4.33727879	4.27605113	6.4140767	4.30508747	6.45763121

7. CONCLUSIONS

The various control schemes considered here are, in effect, continuous hypothesis tests. These hypotheses can be stated as

$H_0 : \mu = T$

against the alternatives

$$\left.\begin{array}{l} H_1 : \mu < T \\ H_1 : \mu > T \end{array}\right\} \text{“Single –sided” schemes}$$

$H_0 : \mu \neq T$ “Two-sided” schemes

In real world, it is frequently unknown whether the process averages μ will change suddenly or gradually. Most of the ARL calculations are based on the one standard deviation schemes under a trend alternative.

If trend is expected in the process average, this prior knowledge will be incorporated in to the design of control procedures while considering sampling intervals.

The data in table [5] and [6] in the case of Weibull distribution suggest that there exists less difference in performance between schemes A, W and C under the trend than under step change conditions. Where as in scheme C with Weibull distribution the lower ARL for slippage of 0.2 to 0.1 standard errors is noted. From table [6], it can be observed that the standard C scheme with Weibull distribution gives somewhat quicker response over the range $0.015 \leq \Delta \leq 0.6$ as compared with schemes of A and W schemes over the range $0.03 \leq \Delta \leq 0.3$. These results are broadly compatible for those relating to step changes, in that, for example with ARL ≅ 6, at Δ = 0.3 for W and C schemes, the process mean must have shifted above two standard errors by the time the trend is detected. Similarly, for A and C shift is about 3 standard error with ARL ≅ 4.9 at Δ= 0.6. For step changes greater than 2.5 σ_e, it is observed that lower ARL for W when compared with C scheme. The same situation is prevailed in the case of slippage greater than 2.5 σ_e. In table [6], the values of Δ ARL gives for further clarification on the point of selection A, W and C alternatives.

REFERENCES

1. Page, E.S., (1954). Continuous inspection schemes. Biometrika, 41,100-114.
2. Rendtel, U. (1990). Cusum schemes with variable sampling intervals and sample sizes. Statistical papers, Springer Verlag 31, 103-118.
3. Johnson, N.L (1966). Cumulative sum control charts and the Weibull Distribution. Technometrics, vol 8,481-491
4. James M. Lucas and Ronald B Crosier (2000). Fast initial Response for Cusum quality control schemes: Give your Cusum head start. Techno metrics, 42 No.1
5. Brook, D and Evans, D.A. (1972). An approach to the probability distribution of Cusum run length, Biomertika, vol 59.
6. Cox, M.A.A. (2009). A comparison of Formulas for the design of cumulative sum control charts. Quality Reliability Engineering, 25,731-737.
7. Koning, A.J. and Does, R. J.M.M. (1988). Cusum charts for preliminary analysis of individual observations. Manuscript, submitted for publication.
8. Sullivan, J.H. and Woodall, W.H. (1996). A control chart for preliminary analysis of individual observations. Journal of quality technology 28, pp.265- 278.
9. Hawkins, D.M. (1977). Testing a sequence of observations for a shift in location. Journal of the American Statistical Association 72, pp.180-186
10. Hinkley, D.V. (1970). Inference about the change point in a sequence of random variables. Biomertika, 57, pp.1-17.
11. Sen, A.and Srivastava, M.S. (1975). On tests for detecting change in mean when variance is unknown. Annuals of the Institute of Statistical Mathematics 27, pp.479-486.
12. Khan R.A. (1978). Wald’s approximations to the average run length in CUSUM procedures. Statist. Plan. and Inf., 2, 63-77.
13. Bissel, A.F. (1984). Estimation of Linear Trend from a Cusum Chart or Tabulation, Journal of Applied Statistics, vol 33.152 – 157.
14. Weibull, W. (1951). A statistical distribution functions of wide applicability. J. Appl. Mech. 18.293-297.
15. Kao, J.H.K. (1959). A graphical estimation of mixed Weibull parameters in life testing electron tubes. Technometrics 1, 389-407.
16. Lieblein, J. and Zelen, M. (1956). Statistical investigation of the fatigue life of deep groove ball bearings. J. of Res. Natl. Bur, Std. 57, 273-315. Literature survey. Journal of quality technology, vol 14

17. Mann, N. (1968). Results on statistical estimation and hypothesis testing with application to the Weibull and extreme value distribution. Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio.
18. Berrettoni, J.N. (1964). Practical applications of the Weibull distribution. Industrial Quality Control, 21, 71-79.

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